Anomalous quasiparticle lifetime in geometric quantum critical metals

arXiv: 2310.07539

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Collaborators

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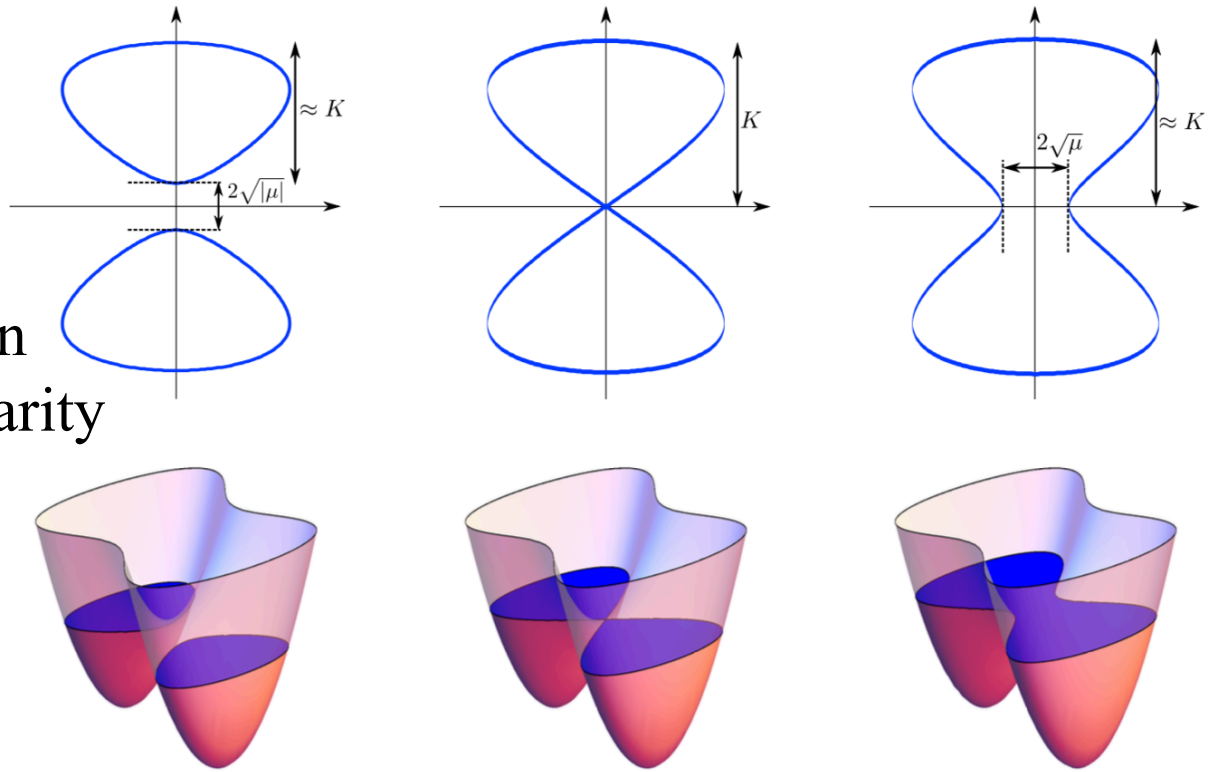
Reference:
Song, Ma, Kallin, Lee, "Anomalous quasiparticle lifetime in geometric quantum critical metals", arXiv: 2310.07539.

Outline

- **Motivation and our main results**
- **Derivations and details of our results**
- **Summary and outlook**

Transition of fermi surface **topology**

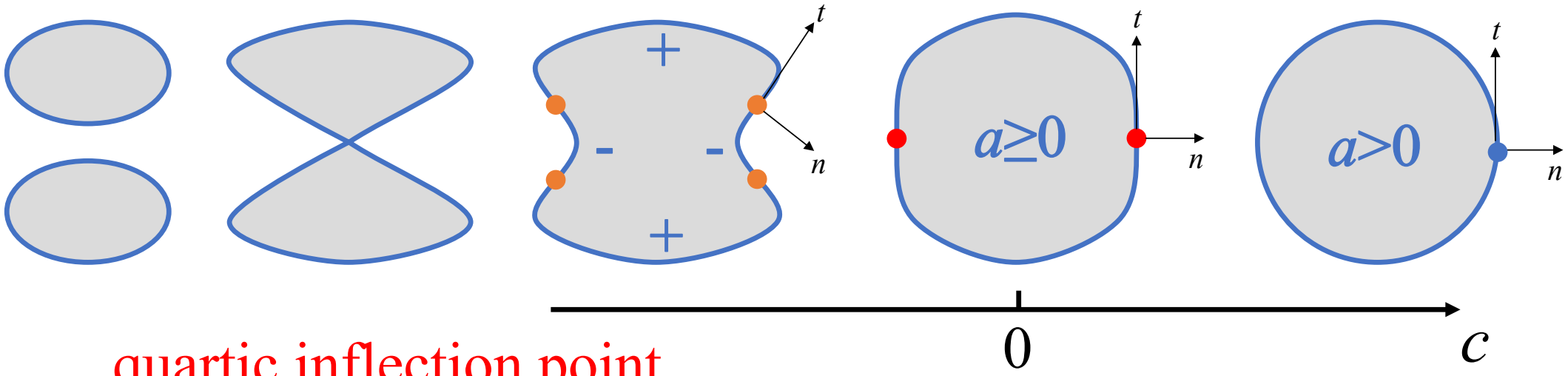
Lifshitz transition
van Hove singularity



$$\xi_{\mathbf{k}} = k_x^2 - k_y^2 - \mu$$

Figure from Ghamari, Lee, Kallin, PRB (2014).

Transition of fermi surface **geometry**



- quartic inflection point

$$E_{\mathbf{K+k}} = v_F k_n + k_t^4$$

$$E_{\mathbf{k}} = k_x^2 + c\, k_y^2 + k_y^4 - \mu$$

- cubic inflection point

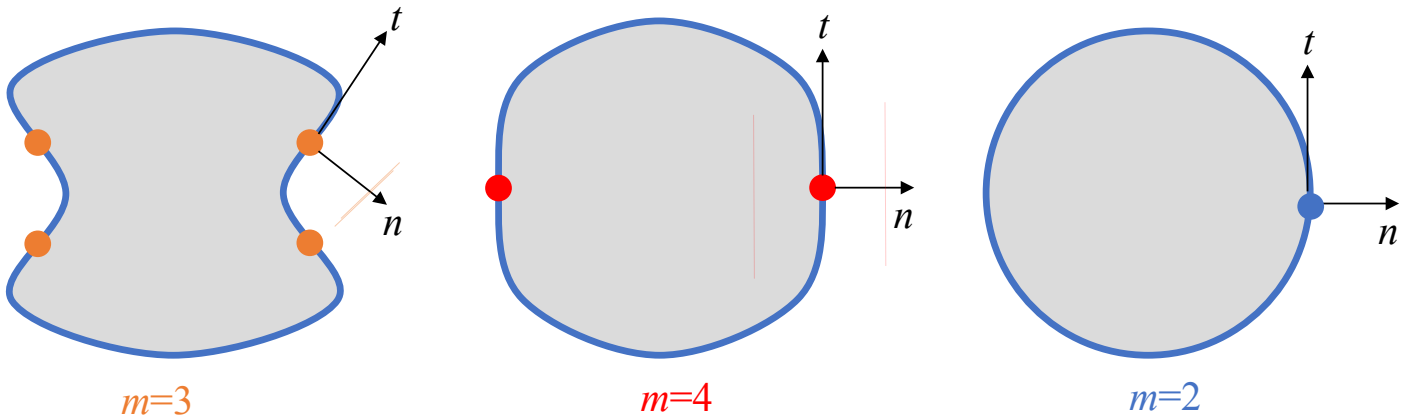
$$E_{\mathbf{K+k}} = v_F k_n + k_t^3$$

- generic point on Fermi Surface

$$E_{\mathbf{K+k}} = v_F k_n + a\, k_t^2 + o(|k_n| + k_t^2)$$

Questions

- Are there any ***qualitatively*** different low energy physics caused by the appearance of Fermi Surface ***inflection*** **points**?
- If yes, can we make any *simple* and *universal* predictions about it?



$$E_{\mathbf{K}+\delta\mathbf{k}} = v_F \delta k_n + (\delta k_t)^m$$

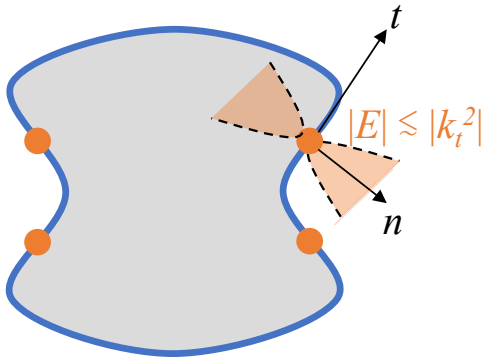
Our main results

Qualitatively different low energy physics due to the Fermi Surface inflection points?

Our results (based on random phase approximation & patch approximation):

- Near **K** with $E_{\mathbf{K}+\delta\mathbf{k}} = v_F \delta k_n + (\delta k_t)^m$, the quasiparticle decay rate $\propto$

$$\left|\Im\Sigma_{\mathbf{k}}\left(E_{\mathbf{k}}\right)\right| \overset{\mathbf{k}\overset{m}{\to}\mathbf{K}}{\propto} \begin{cases} E_{\mathbf{k}}^2 \ln E_{\mathbf{k}}, & m = 2, \\ E_{\mathbf{k}}^{\frac{m}{m-1}}, & m > 2. \end{cases}$$



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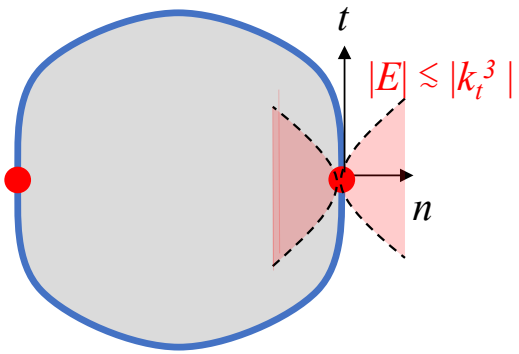

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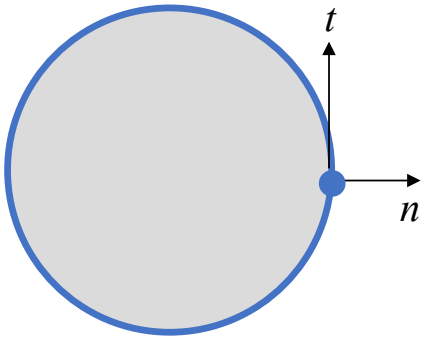

m=2

Our main results

Qualitatively different low energy physics due to the Fermi Surface inflection points?

Our results (based on random phase approximation & patch approximation):

- Near **K** with $E_{\mathbf{K}+\delta\mathbf{k}} = v_F \delta k_n + (\delta k_t)^m$, the electron decay rate $\propto$

$$|\Im\Sigma_{\mathbf{k}}(E_{\mathbf{k}})| \overset{\mathbf{k}\overset{m}{\to}\mathbf{K}}{\propto} \begin{cases} E_{\mathbf{k}}^2 \ln E_{\mathbf{k}}, & m = 2, \\ E_{\mathbf{k}}^{\frac{m}{m-1}}, & m > 2. \end{cases}$$

Outline

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Outline of our calculation

Bare Hamiltonian

$$H = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\xi_{\mathbf{k}}\psi^{\dagger}_{\mathbf{k}}\psi_{\mathbf{k}} + \int \frac{d^2\mathbf{q}}{(2\pi)^2}\int \frac{d^2\mathbf{k}_1}{(2\pi)^2}\int \frac{d^2\mathbf{k}_2}{(2\pi)^2}\frac{V_{\mathbf{q}}}{2}\psi^{\dagger}_{\mathbf{k}_1+\mathbf{q}}\psi^{\dagger}_{\mathbf{k}_2-\mathbf{q}}\psi_{\mathbf{k}_2}\psi_{\mathbf{k}_1}, \; V_{\mathbf{q}} = \frac{e^2}{|\mathbf{q}|}.$$

Self-consistent equations

$$G(\mathbf{p},\varepsilon) = \frac{1}{-\varepsilon(1+i0^+) + \xi_{\mathbf{p}} + \Sigma(\mathbf{p},\varepsilon)},$$

$$\Sigma(\mathbf{p},\varepsilon) = -i\int_{\mathbf{q},\omega} \mathcal{V}(\mathbf{q},\omega)\, G(\mathbf{p}-\mathbf{q},\varepsilon-\omega),$$

$$\mathcal{V}(\mathbf{q},\omega) = \frac{1}{V_{\mathbf{q}}^{-1} - \Pi(\mathbf{q},\omega)},$$

$$\Pi(\mathbf{q},\omega) = -i\int_{\mathbf{p},\varepsilon} G(\mathbf{p},\varepsilon)\, G(\mathbf{p}+\mathbf{q},\varepsilon+\omega).$$

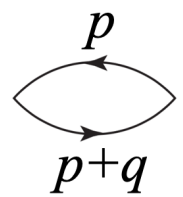
$$E_{\mathbf{p}} \Longrightarrow \mathrm{Im}\Sigma(\mathbf{p}, E_{\mathbf{p}}) \sim \frac{1}{\tau_{\mathbf{p}}}$$

Dyson's equation for Green's function

leading order of self-energy

screened interaction

polarization in RPA



Self-consistency computation

$$G(\mathbf{p},\varepsilon) = \frac{1}{-\varepsilon(1+i0^+)+\xi_{\mathbf{p}}+\Sigma(\mathbf{p},\varepsilon)} \simeq \frac{Z_{\mathbf{p}}}{-\varepsilon+E_{\mathbf{p}}-i\,\mathrm{sgn}(\varepsilon)\,\tau_{\mathbf{p}}^{-1}}$$

Valid if $\tau_{\mathbf{p}}^{-1} \ll |E_{\mathbf{p}}|$

$$G(\mathbf{p},\varepsilon) \approx \frac{1}{-\varepsilon(1+i0^+)+E_{\mathbf{p}}}$$

$$E_{\mathbf{p}} = \xi_{\mathbf{p}} + \mathrm{Re}\Sigma(\mathbf{p},E_{\mathbf{p}})$$
$$\tau_{\mathbf{p}}^{-1} \sim \left|\mathrm{Im}\,\Sigma\left(\mathbf{p},E_{\mathbf{p}}\right)\right|$$

$$\Sigma(\mathbf{p},\varepsilon) = -i\int_{\mathbf{q},\omega} \mathcal{V}(\mathbf{q},\omega)\,G(\mathbf{p}-\mathbf{q},\varepsilon-\omega)$$

$$\Pi(\mathbf{q},\omega) = -i\int_{\mathbf{p},\varepsilon} G(\mathbf{p},\varepsilon)\,G(\mathbf{p}+\mathbf{q},\varepsilon+\omega)$$

$$\mathcal{V}(\mathbf{q},\omega) = \frac{1}{V_{\mathbf{q}}^{-1}-\Pi(\mathbf{q},\omega)}$$

Step 1: compute $\Pi(q, \omega)$

$$\Pi(\mathbf{q}, \omega) = -i \int_{\mathbf{p},\varepsilon} G(\mathbf{p}, \varepsilon) G(\mathbf{p}+\mathbf{q}, \varepsilon+\omega) = \int_{\mathbf{p}} \frac{\Theta(-E_{\mathbf{p}+\mathbf{q}}) - \Theta(-E_{\mathbf{p}})}{-\omega(1+i0^+) + E_{\mathbf{p}+\mathbf{q}} - E_{\mathbf{p}}}$$

For small $\omega \to 0$,

$$\Pi(\mathbf{q}, 0) = \int_{\mathbf{p}} \frac{\Theta(-E_{\mathbf{p}+\mathbf{q}}) - \Theta(-E_{\mathbf{p}})}{E_{\mathbf{p}+\mathbf{q}} - E_{\mathbf{p}}} = -\int_{\mathbf{p}} \delta\left(E_{\mathbf{p}}\right)$$

density of quasiparticles at Fermi energy

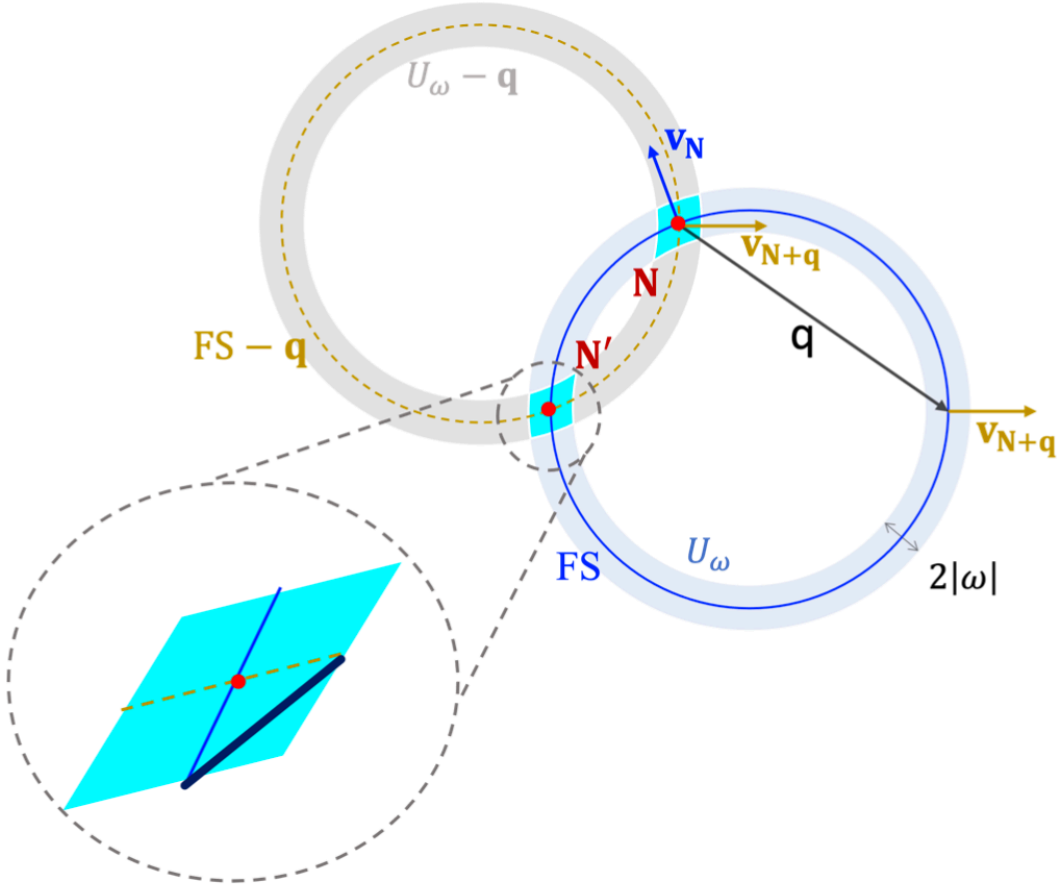
$$\mathrm{Im}\,\Pi(\mathbf{q}, \omega) = \pi\,\mathrm{sgn}(\omega) \int_{\mathbf{p}} \left(\Theta(-E_{\mathbf{p}+\mathbf{q}}) - \Theta(-E_{\mathbf{p}})\right) \delta\left(E_{\mathbf{p}+\mathbf{q}} - E_{\mathbf{p}} - \omega\right) \sim \sum_{\mathbf{N} \in \mathcal{N}_{-\mathbf{q}}} \frac{-|\omega|}{|\mathbf{v}_{\mathbf{N}+\mathbf{q}} \times \mathbf{v}_{\mathbf{N}}|}$$

$$\Pi(\mathbf{q}, \omega) \sim -1 - i \sum_{\mathbf{N} \in \mathcal{N}_{-\mathbf{q}}} \frac{|\omega|}{|\mathbf{v}_{\mathbf{N}+\mathbf{q}} \times \mathbf{v}_{\mathbf{N}}|}$$

for the purpose of determining scaling behavior

Step 1: compute $\Pi(q, \omega)$

$$\mathrm{Im}\,\Pi(\mathbf{q},\omega) = \pi\,\mathrm{sgn}(\omega)\int_{\mathbf{p}}\left(\Theta(-E_{\mathbf{p}+\mathbf{q}})-\Theta(-E_{\mathbf{p}})\right)\delta\left(E_{\mathbf{p}+\mathbf{q}}-E_{\mathbf{p}}-\omega\right) \sim \sum_{\mathbf{N}\in\mathcal{N}_{-\mathbf{q}}} \frac{-|\omega|}{|\mathbf{v}_{\mathbf{N}+\mathbf{q}}\times\mathbf{v}_{\mathbf{N}}|}$$



$$\mathcal{N}_{-\mathbf{q}} := \{\mathbf{N}\,|\,\mathbf{N}\in(\mathrm{FS}-\mathbf{q})\cap\mathrm{FS}\}$$

For inflection point N and small q ,

$$E_{\mathbf{N}+\mathbf{q}} = v_F q_\perp + a_m q_\parallel^m,$$

$$\frac{|\omega|}{|\mathbf{v}_{\mathbf{N}+\mathbf{q}}\times\mathbf{v}_{\mathbf{N}}|} \sim \frac{|\omega|}{\left|q_\parallel^{m-1}\right|}.$$

Step 2: compute $\mathcal{V}(q,\omega)$

For the purpose of determining scaling behavior, $\Pi(\mathbf{q},\omega) \sim -1 - i\sum_{\mathbf{N}\in\mathcal{N}_{-\mathbf{q}}} \frac{|\omega|}{|\mathbf{v}_{\mathbf{N}+\mathbf{q}} \times \mathbf{v}_{\mathbf{N}}|}$

$$\mathcal{V}(\mathbf{q},\omega) = \frac{1}{V_{\mathbf{q}}^{-1} - \Pi(\mathbf{q},\omega)} \sim \frac{1}{\frac{|\mathbf{q}|}{2\pi e^2} + 1 - i\,\mathrm{Im}\Pi(\mathbf{q},\omega)} \sim \frac{1}{1 - i\,\mathrm{Im}\,\Pi(\mathbf{q},\omega)}$$

Coulomb potential is “screened”.

Step 3: compute imaginary part of $\Sigma(p, \varepsilon)$

Use the spectral representation $f^R(\mathbf{p}, \varepsilon) = \int \frac{d\varepsilon'}{\pi} \frac{\mathrm{Im}\, f^R(\mathbf{p}, \varepsilon')}{-(\varepsilon + i0^+) + \varepsilon'}$ for the retarded Green's functions

$$\Sigma(\mathbf{p}, \varepsilon) = -i \int_{\mathbf{q}, \omega} \mathcal{V}(\mathbf{q}, \omega)\, G(\mathbf{p} - \mathbf{q}, \varepsilon - \omega)$$

$$\Sigma(\mathbf{p}, \varepsilon) = \int \frac{d^2 q}{(2\pi)^2} \int \frac{d\omega'}{\pi} \int \frac{d\varepsilon'}{\pi} \frac{\mathrm{Im}\, \mathcal{V}^R(\mathbf{q}, \omega')\, \mathrm{Im}\, G^R(\mathbf{p} - \mathbf{q}, \varepsilon')}{-\varepsilon(1 + i0^+) + \varepsilon' + \omega'} \left[\Theta(\varepsilon') - \Theta(-\omega')\right]$$

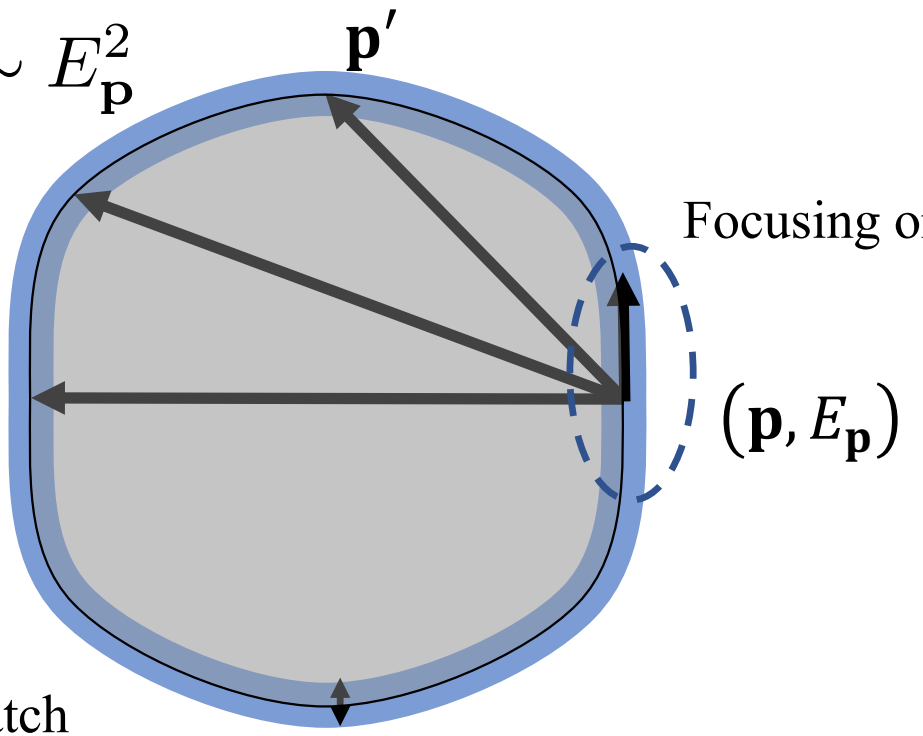
$$\mathrm{Im}\, \Sigma(\mathbf{p}, \varepsilon) \approx -\mathrm{sgn}(\varepsilon) \int \frac{d^2 p'}{(2\pi)^2} \left|\mathrm{Im}\, \mathcal{V}^R\left(\mathbf{p} - \mathbf{p}', \varepsilon - E_{\mathbf{p}'}\right)\right| \Theta\left(E_{\mathbf{p}'}\left(\varepsilon - E_{\mathbf{p}'}\right)\right)$$

$$\tau_{\mathbf{p}}^{-1} \sim \left|\mathrm{Im}\, \Sigma\left(\mathbf{p}, E_{\mathbf{p}}\right)\right| \sim \int_{\mathbf{p}'} \left|\mathrm{Im}\, \mathcal{V}\left(\mathbf{p} - \mathbf{p}', E_{\mathbf{p}} - E_{\mathbf{p}'}\right)\right| \Theta\left(E_{\mathbf{p}'}\left(E_{\mathbf{p}} - E_{\mathbf{p}'}\right)\right)$$

Step 4: compute τ_p

$$\frac{1}{\tau_{\mathbf{p}}} \sim \int_{\mathbf{p}'} \left|\mathrm{Im}\,\mathcal{V}\left(\mathbf{p}-\mathbf{p}', E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right| \Theta\left(E_{\mathbf{p}'}\left(E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right) \sim \int_{\mathbf{p}'} \left| \mathrm{Im}\frac{\Theta\left(E_{\mathbf{p}'}\left(E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right)}{1+i\frac{|\omega|}{|\mathbf{v}_{\mathbf{p}}\times\mathbf{v}_{\mathbf{p}'}|}} \right|$$

The decay rate due to scattering between unparalleled patches: $\frac{1}{\tau_{\mathbf{p}}^{\nparallel}} \sim E_{\mathbf{p}}^2$



$$\frac{1}{\tau_{\mathbf{p}}^{\parallel}} \sim \begin{cases} \frac{1}{a_2} E_{\mathbf{p}}^2 \ln\left|\frac{1}{E_{\mathbf{p}}}\right|, & n=2 \\ a_n^{-\frac{1}{n-1}} E_{\mathbf{p}}^{\frac{n}{n-1}}, & n>2 \end{cases}$$

n: order of inflection point on a parallel patch

Step 4: compute τ_p

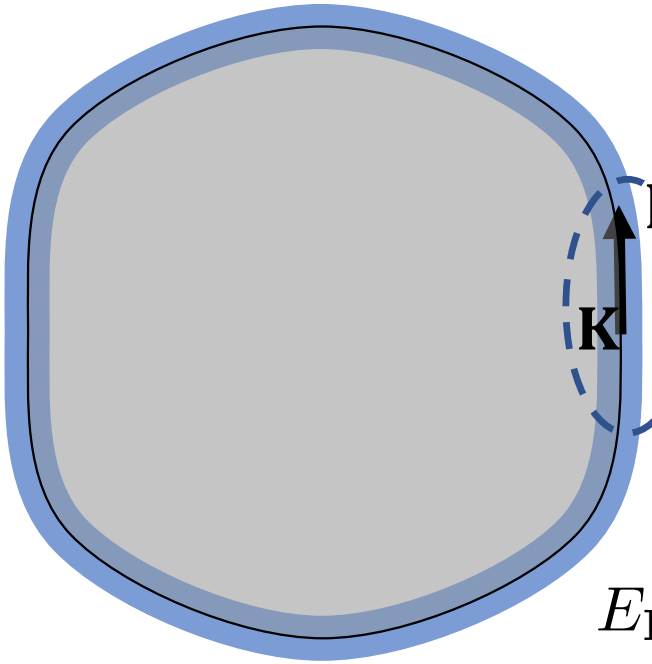
$$\frac{1}{\tau_{\mathbf{p}}} \sim \int_{\mathbf{p}'} \left|\mathrm{Im}\,\mathcal{V}\left(\mathbf{p}-\mathbf{p}', E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right| \Theta\left(E_{\mathbf{p}'}\left(E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right) \sim \int_{\mathbf{p}'} \left|\mathrm{Im}\frac{\Theta\left(E_{\mathbf{p}'}\left(E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right)}{1+i\frac{|\omega|}{\left|\mathbf{v}_{\mathbf{p}}\times\mathbf{v}_{\mathbf{p}'}\right|}}\right|$$

An **exact** computation is possible for scattering within a single patch:

$$\frac{1}{\tau_{\mathbf{p}}^{\parallel}} \sim \int_{E_{\mathbf{p}'},k'_{\parallel}} \left|\mathrm{Im}\,\frac{\Theta\left(E_{\mathbf{p}'}\left(E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right)}{1-i\,\frac{1}{a_n}\frac{E_{\mathbf{p}}-E_{\mathbf{p}'}}{\left|k_{\parallel}'^{n-1}\right|}}\right|$$

$$\sim \begin{cases} \frac{1}{a_2}E_{\mathbf{p}}^2 \ln\frac{1}{E_{\mathbf{p}}}, & n=2 \\ \frac{1}{a_n^{1/(n-1)}}E_{\mathbf{p}}^{\frac{n}{n-1}}, & n>2 \end{cases}$$

for $\mathbf{p} = \mathbf{K} + (k_\perp, 0)$.



$$E_{\mathbf{K}+\mathbf{k}} = v_F k_\perp + a_n k_\parallel^n$$

Determine the crossover

$$\frac{1}{\tau_{\mathbf{p}}} \sim \int_{\mathbf{p}'} \left|\mathrm{Im}\,\mathcal{V}\left(\mathbf{p}-\mathbf{p}', E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right| \Theta\left(E_{\mathbf{p}'}\left(E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right) \sim \int_{\mathbf{p}'} \left|\mathrm{Im}\frac{\Theta\left(E_{\mathbf{p}'}\left(E_{\mathbf{p}}-E_{\mathbf{p}'}\right)\right)}{1+i\frac{|\omega|}{\left|\mathbf{v}_{\mathbf{p}}\times\mathbf{v}_{\mathbf{p}'}\right|}}\right|$$

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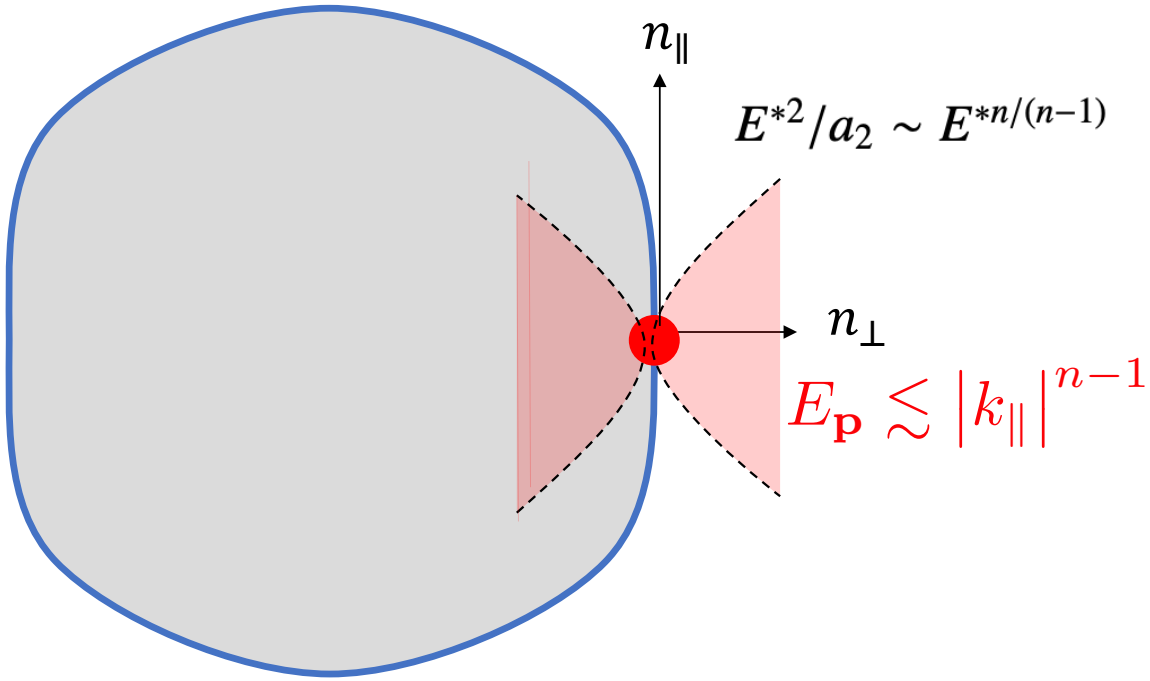
$$\sim \begin{cases} \frac{1}{a_2}E_{\mathbf{p}}^2 \ln\frac{1}{E_{\mathbf{p}}}, & n=2 \\ \frac{1}{a_n^{1/(n-1)}}E_{\mathbf{p}}^{\frac{n}{n-1}}, & n>2 \end{cases}$$



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Summary

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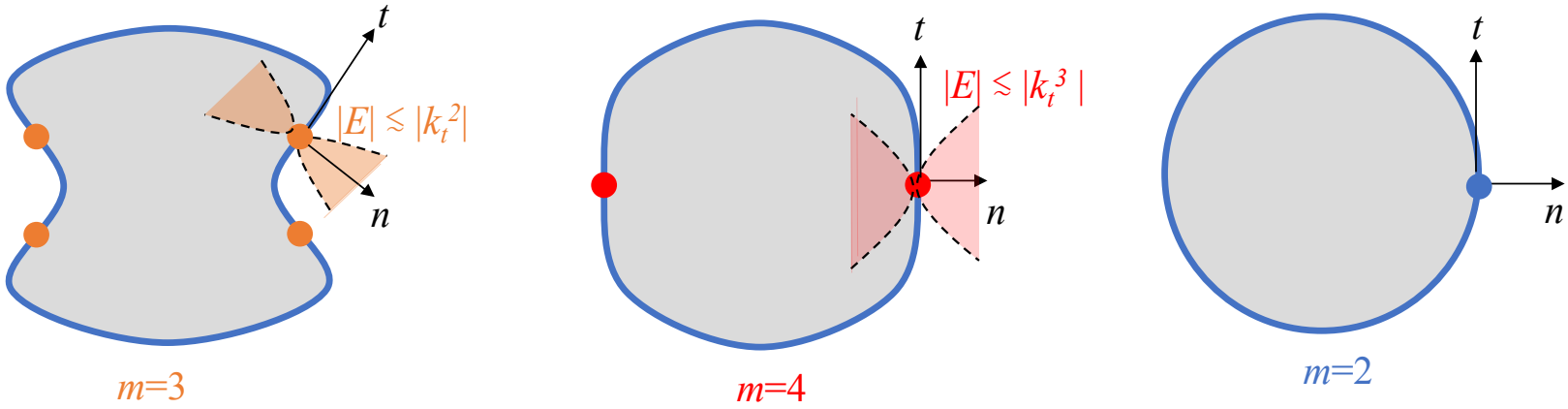
Our results (based on random phase approximation & patch approximation):

- Near **K** with $E_{\mathbf{K}+\delta\mathbf{k}} = v_F \delta k_n + (\delta k_t)^m$, the **quasiparticle decay rate** $\propto$

$$\frac{1}{\tau_{\mathbf{p}}} \sim \begin{cases} E_{\mathbf{p}}^2 \ln \left| \frac{1}{E_{\mathbf{p}}} \right|, & m = 2 \\ E_{\mathbf{p}}^{\frac{m}{m-1}}, & m > 2 \end{cases}$$



Outlook

- The leading **non-analytic** correction to the **renormalized dispersion** $E_{\mathbf{p}}$ at the inflection points.

- How the geometric quantum criticality manifests itself in **collective modes** that describe fluctuations of Fermi surface shape?

- Try to explicitly test our predictions by photoemission spectroscopy.

Thank you for your attention!